Answer Keys of Term-I Syllabus

Class: 9th

Subject: Mathematics

Contents:
1. Quadrilaterals
2. Constructions
3. Statistics

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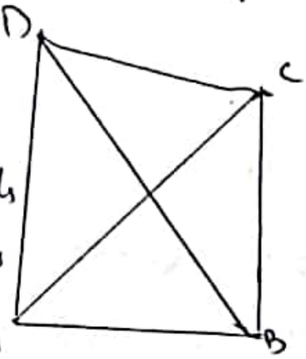
Quadrilaterals

Quadrilateral: The word 'quad' means four and the word 'lateral' means sides. Thus, a plane figure bounded by four line segments is called a quadrilateral.

The line segments AC and BD are called the diagonals of quad. ABCD. So, diagonals are the line segments joining the opposite vertices.



Consecutive or adjacent sides: Two sides of quad. are consecutive or adjacent, if they have a common vertex. In the given fig. AB & BC; BC & CD; CD & AD; AD & AB are adjacent sides.

Opposite sides: Two sides of a quad. are opposite sides; if they have no common end point (vertex). In given fig. AB & CD; AD & BC are opposite sides.

Consecutive or adjacent angles: Two angles are said to be consecutive if they have a common arm.

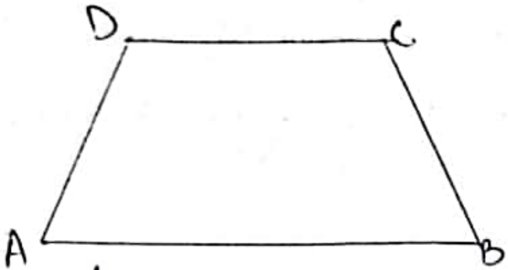
$\angle A$ & $\angle B$; $\angle B$ & $\angle C$; $\angle C$ & $\angle D$; $\angle D$ & $\angle A$ are consecutive angles.

Opposite Angles: Two angles of a quadrilateral are said to be opposite angles if they donot have a common arm.

$\angle A$ & $\angle C$; $\angle C$ & $\angle D$ are two pairs of opposite angles of quad. ABCD

Various types of Quadrilaterals

Trapezium: A quadrilateral having exactly one pair of parallel sides, is called a trapezium.



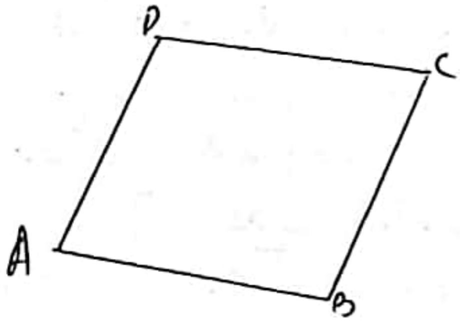
ABCD is a trapezium in which AB ∥ DC

Parallelogram: A quadrilateral is a parallelogram if its both pairs of opposite sides are parallel.

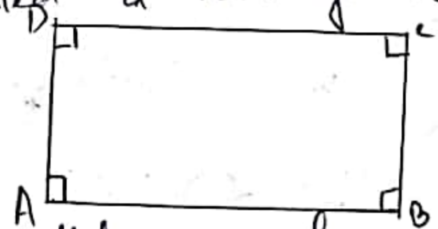
ABCD is a parallelogram in which AB ∥ DC, AD ∥ BC and AB = CD and AD = BC

Rhombus: A ∥gm having all sides equal is called a rhombus.

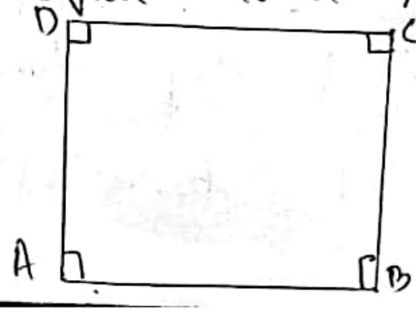
Thus, a ∥gm ABCD is a rhombus if AB = BC = CD = DA



Rectangle: A parallelogram whose each angle is a right angle, is called a rectangle. i.e, $\angle A = \angle B = \angle C = \angle D = 90^\circ$



Square: A parallelogram having all sides equal and each angle equal to a right angle, is called a square.

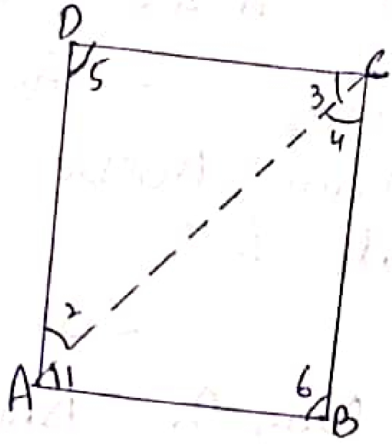


AB = BC = CD = DA &

$\angle A = \angle B = \angle C = \angle D = 90^\circ$

Angle Sum Property Of a Quadrilateral

The Sum of the four angles of a Quadrilateral is 360°.



Given: Quadrilateral ABCD

To Prove: $\angle A + \angle B + \angle C + \angle D = 360°$

Construction: Join AC

Proof: In $\triangle ABC$, we have

$\angle 1 + \angle 4 + \angle 6 = 180°$ — ① (Angle Sum Property of $\triangle$)

In $\triangle ACD$, we have

$\angle 2 + \angle 3 + \angle 5 = 180°$ — ②

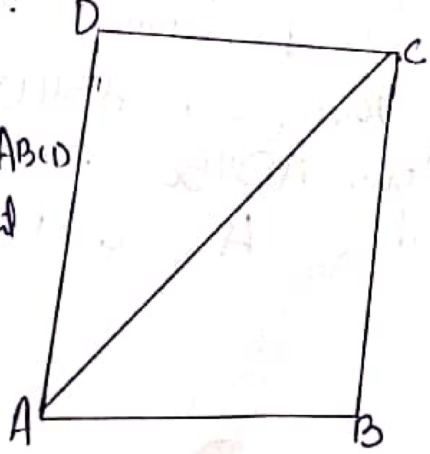
Adding ① & ②, we get

$(\angle 1 + \angle 2) + (\angle 3 + \angle 4) + \angle 5 + \angle 6 = 180° + 180°$

$\Rightarrow \angle A + \angle C + \angle D + \angle B = 360°$

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360°$

Theorem 7.1: A diagonal of a parallelogram divides it into two Congruent triangles.



Given: A Parallelogram ABCD

To Prove: Diagonal AC of a ||gm ABCD divides it into two Congruent triangles.

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Proof: Since ABCD is a Parallelogram. Therefore, AB ∥ DC and AD ∥ BC

Now, AD ∥ BC and transversal AC intersects them at A & C respectively.

$\therefore$ $\angle DAC = \angle BCA$ — (i) {Alt. interior angles

Again, AB ∥ DC and transversal AC intersects them at A & C respectively. Therefore,

$\angle BAC = \angle DCA$ — (ii) {Alt. int. $\angle$'s

Now, In ΔABC & ΔCDA, we have

$\angle BCA = \angle DAC$ {from (i)

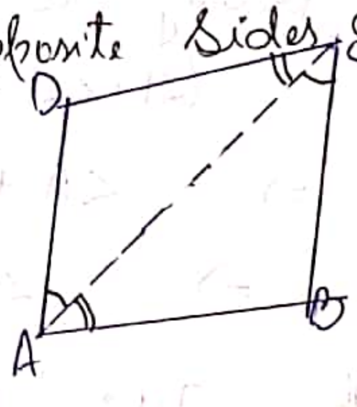
$AC = AC$ {Common side

$\angle BAC = \angle DCA$ {from (ii)

$\Rightarrow$ $\Delta ABC \cong \Delta CDA$ $\longrightarrow$ ASA Congruence Criterion

Hence proved

Theorem 7.2: In a Parallelogram, opposite sides are equal.



Given: A Parallelogram ABCD

To Prove: AB = CD and DA = BC

Construction: Join AC

Proof: Since ABCD is a Parallelogram.

Therefore, AB ∥ DC & AD ∥ BC

Now, AD ∥ BC and transversal AC intersects them at A and C respectively.

$\therefore \angle DAC = \angle BCA$ .... (i) {Alternate interior ∠'s

Again, AB ∥ DC and transversal AC intersects them at A & C respectively

$\therefore \angle BAC = \angle DCA$ .... (ii) {Alternate interior ∠'s

Now, In ΔADC and ΔCBA, we have

$\angle DAC = \angle BCA$ {from (i)

$AC = AC$ {Common Side

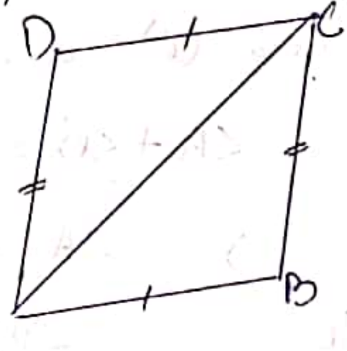
and $\angle DCA = \angle BAC$ {from (ii)

So, by ASA Criterion of Congruence

$\Delta ADC \cong \Delta CBA$

$\Rightarrow AD = CB$ and $DC = BA$ { C.P.C.T

Theorem 7.3: If each pair of opposite sides of a quadrilateral is equal, then it is a Parallelogram.



Given: A Quad ABCD in which $AB = CD$ & $AD = BC$

To Prove: Quad. ABCD is a Parallelogram

Proof: In ΔADC & ΔCBA, we have

$AB = CD$ {Given

$AC = AC$ {Common Side

$AD = BC$ {Given

So, by SSS Criterion of Congruence

$\Delta ADC \cong \Delta CBA$

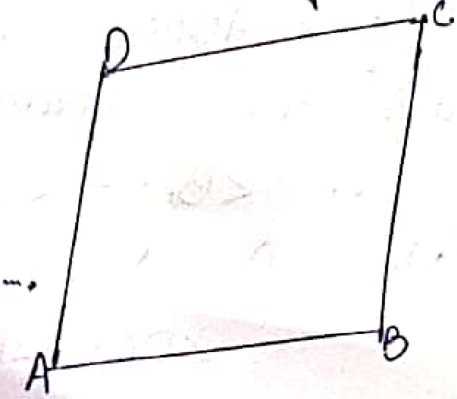
$\Rightarrow \angle DAC = \angle BCA$ {C.P.C.T

$\Rightarrow AD \parallel BC$

& $\angle BAC = \angle DCA$ {C.P.C.T

$\Rightarrow AB \parallel CD$ $\Rightarrow$ ABCD is a Parallelogram

Theorem 7.4: In a Parallelogram, Opposite angles are equal.



Given: A Parallelogram ABCD

To Prove: $\angle A = \angle C$ & $\angle B = \angle D$

Proof: Since ABCD is a Parallelogram.

Therefore,

$AB \parallel DC$ and $AD \parallel BC$

Now, $AB \parallel DC$ and transversal AD intersects them at A and D respectively.

$\therefore \quad \angle A + \angle D = 180^\circ \;....(i)$ { $\because$ Sum of Consecutive interior angles is 180°

Again, $AD \parallel BC$ and DC intersects them at D and C respectively

$\therefore \quad \angle D + \angle C = 180^\circ \;....(ii)$ { $\because$ Sum of Consecutive interior angles is 180°

From (i) and (ii), we get

$$\angle A + \cancel{\angle D} = \cancel{\angle D} + \angle C$$

$$\Rightarrow \quad \angle A = \angle C$$

Illy, $\quad \angle B = \angle D$

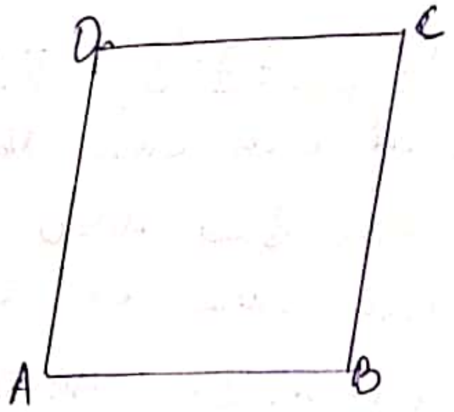
Hence, $\angle A = \angle C$ and $\angle B = \angle D$

Theorem 7.5: If in a Quadrilateral each pair of Opposite angles is equal, then it is a Parallelogram.

Given: A Quad ABCD in which $\angle A = \angle C$ and $\angle B = \angle D$

To Prove: Quad. ABCD is a Parallelogram

Proof: Since, the sum of angles of a Quad. is $360°$



$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \angle A + \angle B + \angle A + \angle B = 360°$ { $\because \angle A = \angle C$ & $\angle B = \angle D$

$\Rightarrow 2\angle A + 2\angle B = 360°$

$\Rightarrow 2(\angle A + \angle B) = 360°$

$\Rightarrow \angle A + \angle B = \cancel{\frac{360°}{2}}\ 180°$

$\Rightarrow \angle A + \angle B = 180°$ (Co-interior angles)

$\Rightarrow AD \parallel BC$

IIly we can prove $AB \parallel DC$

Now In Quad. ABCD

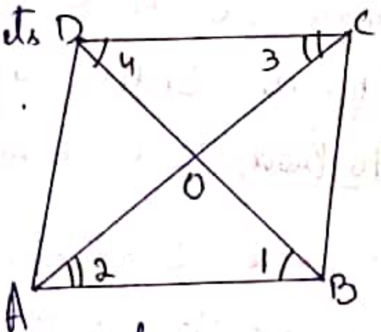
$AB \parallel CD$

& $AD \parallel BC$

$\Rightarrow$ Quadrilateral ABCD is a Parallelogram

Theorem 7.6: The diagonals of a Parallelogram bisect each other

Given: A Parallelogram ABCD such that its diagonals AC and BD intersect at O.



To Prove: $OA = OC$ and $OB = OD$

Proof: In ΔABO & ΔCDO

$\angle 1 = \angle 4$ { $\because AB \parallel DC$ & BD is a transversal Alt. int $\angle$'s

$AB = CD$ { Opposite sides of llgm

$\angle 2 = \angle 3$ { Alt. int. $\angle$'s

$\therefore$ By ASA Congruence Criterion

$\Delta ABO \cong \Delta CDO$

$\Rightarrow OA = OC$ } C.P.C.T

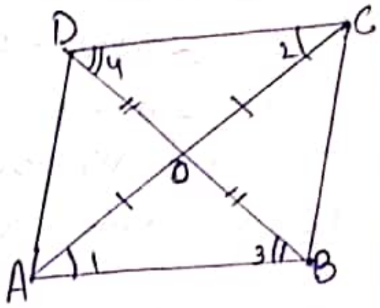
& $OB = OD$

Hence Proved

Theorem 7.7: If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Given: Quad. ABCD in which diagonals bisect each other i.e, OA = OC and OB = OD

To Prove: Quad. ABCD is a Parallelogram.



Proof: In ΔAOB & ΔCOD, we have

$OA = OC$ {Given

$\angle AOB = \angle COD$ {vertically opp. ∠'s

$OB = OD$

∴ By SAS Congruence Criterion

$\Delta AOB \cong \Delta COD$

$\angle 1 = \angle 2$ & $\angle 3 = \angle 4$ {C.P.C.T

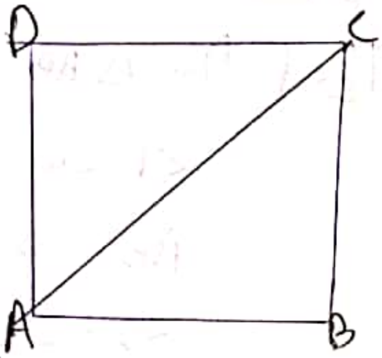
=> $AB \parallel CD$ & $AD \parallel BC$

Therefore, ABCD is a Parallelogram.

Theorem 7.8:- A Quad. is a ||gm if its one pair of opposite sides are parallel and equal.

Given A Quad. ABCD in which $AB = CD$ & $AB \parallel CD$

To Prove:- Quad. ABCD is a ||gm



Proof:- In ΔABC & ΔCDA, we have

$AB = DC$ {Given

$AC = AC$ {Common side

$\angle BAC = \angle DCA$ {alt. int. ∠'s

∴ $\Delta ABC \cong \Delta CDA$ {SAS Criterion

=> $\angle BCA = \angle DAC$ {C.P.C.T

=> alt. int. ∠'s are equal

=> $AD \parallel BC$

Thus, $AB \parallel CD$ & $AD \parallel BC$

Hence Quad. ABCD is a ||gm

ABCD be a quadrilateral

Theorem 7.9

The Mid-Point Theorem

Statement: The line segment joining the mid-points of any two sides of a triangle is parallel to the thired side and equal to half of it.

Given: A ΔABC in which D and E are the mid-points of sides AB and AC respectively. DE is joined.

To prove: $DE \parallel BC$ and $DE = \frac{1}{2} BC$

CONSTRUCTION: Produce the line segment DE to F, such that $DE = EF$. join FC.

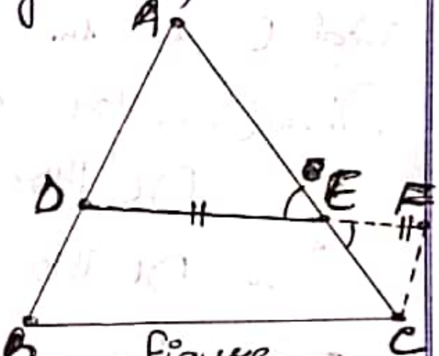
PROOF: In Δs AED and CEF, we have

$AE = CE$ [$\because$ E is the mid-point of AC]

$\angle AED = \angle CEF$ [vertically opposite angles]

and, $DE = EF$ [By construction]

So, by SAS criterion of congruence, we have



figure

$\Delta AED \cong \Delta CEF$

$\Rightarrow$ $AD = CF$ [c.p.c.t] $\rightarrow$ (i)

and, $\angle ADE = \angle CFE \rightarrow$ (ii)

Now, D is the mid-point of AB

$\Rightarrow AD = DB$

$\Rightarrow DB = CF$ [from (i) $AD = CF$] $\rightarrow$ (iii)

Now, DF intersects AD & FC at D & F resp such that

$\angle ADE = \angle CFE$ [from (ii)]

ie, alternate interior angles are equal

$\therefore AD \parallel FC$

$\Rightarrow DB \parallel CF \rightarrow$ (iv)

From (iii) and (iv), we find that DBCF is a quadrilateral such that one pair of sides are equal & parallel.

∴ DBCF is a parallelogram.

⇒ $DF \parallel BC$ and $DF = BC$ [∵ opp. sides of a ||gm are equal and parallel]

But, D, E, F are collinear and $DE = EF$.

∴ $DE \parallel BC$ and $DE = \frac{1}{2} BC$

Converse Of Mid-Point Theorem

Statement: The line drawn through the mid-point of one side of a triangle, parallel to another side, intersects the third side at its mid-point.

Given: ΔABC in which D is the mid-point of AB and $DE \parallel BC$

To Prove: E is the mid-point of AC

Proof: we have to prove that E is the mid-point of AC. If possible, let E be not the mid-point of AC. let E' be the mid-point of AC. Join DE'.

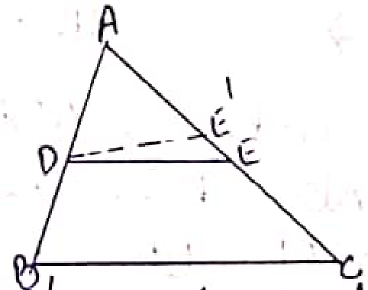
Now, In ΔABC, D is the mid-point of AB (Given)

and E' is the mid point of AC.

Therefore, By mid point theorem, we have

$DE' \parallel BC$ ... (i)

Also $DE \parallel BC$ ... (ii)



⇒ $DE \parallel DE'$

⇒ Two intersecting lines, DE and DE' are both parallel to line BC. which is a Contradiction.

So, our Supposition is wrong.

Hence, E is the mid point of AC.

Exercise 8.1

Q1. let the first angle be $3x$

2nd angle be $5x$

3rd angle be $9x$

& 4th angle be $13x$

Now,

$$3x + 5x + 9x + 13x = 360^\circ \quad \{ \text{A.S.P of a quad.}$$

$$30x = 360^\circ$$

$$\Rightarrow x = \frac{\cancel{360}^{12}}{\cancel{30}}$$

$$x = 12$$

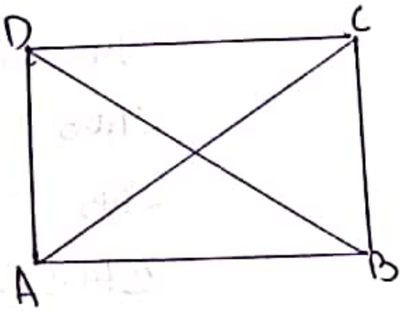
$\therefore$ first angle $= 3x = 3 \times 12 = 36^\circ$

2nd angle $= 5x = 5 \times 12 = 60^\circ$

3rd angle $= 9x = 9 \times 12 = 108^\circ$

& 4th angle $= 13x = 13 \times 12 = 156^\circ$

Q2. In ||gm ABCD, $AC = BD$



In $\triangle ABC$ and $\triangle BAD$

$AB = BA$ {Common side

$BC = AD$ {opp. sides of ||gm

$AC = BD$ {Given

$\therefore \triangle ABC \cong \triangle BAD$ {SSS Congruence Criterion

$\Rightarrow \angle ABC = \angle BAD$ {C.P.C.T

i.e, $\angle A = \angle B$

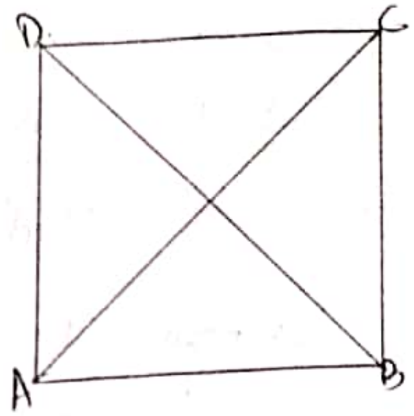
$\angle B = \angle D$ & $\angle A = \angle C$ {opp. $\angle$s of ||gm

$\Rightarrow \angle A = \angle B = \angle C = \angle D = 90^\circ$

Hence ABCD is a rectangle.

Q3: It is given that diagonals bisect each other at rt. <'s i.e

OA = OC & OB = OD



In Δ AOB & Δ AOD, we have

AO = AO (Common side)

OB = OD (Given)

$\angle AOB = \angle AOD$ {each = 90°

$\Rightarrow$ $\Delta AOB \cong \Delta AOD$

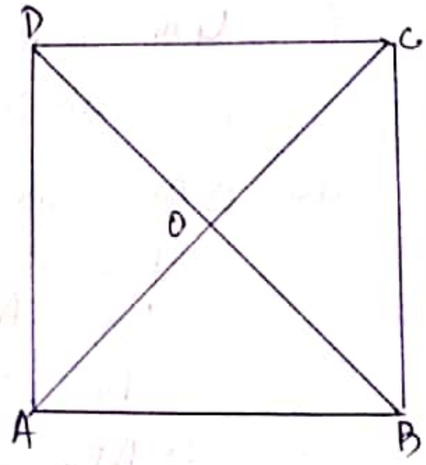
$\Rightarrow$ AB = AD

IIIly AB = BC & AD = CD

$\Rightarrow$ AB = BC = CD = AD

$\Rightarrow$ ABCD is a rhombus

Q4: Diagonals AC and BD of the square ABCD intersect each other at O.



In ΔAOB and ΔCOD,

AB = CD (Sides of a square)

$\angle AOB = \angle COD$ (Vertically opposite <'s)

$\angle ABO = \angle CDO$ (alt. <'s)

$\Rightarrow$ $\Delta AOB \cong \Delta COD$

$\Rightarrow$ OA = OC

IIIly, OB = OD

$\Rightarrow$ AC and BD bisect each other at O.

Now, In Δ AOB & Δ COB

OA = OC {Proved above

OB = OB {Common side

AB = BC {Sides of a square

$\therefore$ $\Delta AOB \cong \Delta COB$ {SSS Cong.

$\Rightarrow$ $\angle AOB = \angle COB$ {C.P.C.T

$\angle AOB + \angle COB = 180^\circ$ {L.P

$\angle AOB + \angle AOB = 180$

$2\angle AOB = 180^\circ$

$\Rightarrow \angle AOB = \frac{180}{2} = 90^\circ$

$\Rightarrow \angle AOB = \angle COB = 90^\circ$

IIIly $\angle AOD = \angle COD = 90^\circ$

Hence, AC & BD bisect each other at rt. <'s

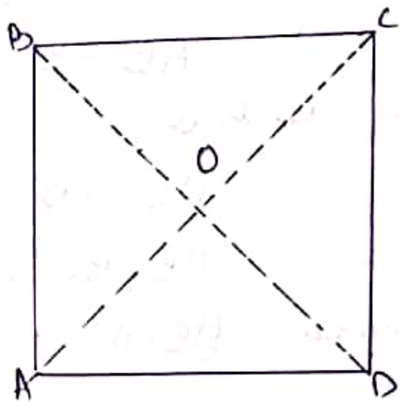
5. let ABCD be a quadrilateral such that its diagonals AC and BD are equal and bisect each other at right angle. we have to prove that it is a square. For this, we will prove that

$\angle A = \angle B = \angle C = \angle D = 90^\circ$ and $AB = BC = CD = DA$

Since diagonals AC and BD of quadrilateral ABCD bisect each other. Therefore, it is a parallelogram (Theorem 7.7)

So, $AB = DC$

& $BC = AD$



Now, In ΔAOB and ΔCOB, we have

$OA = OC$ {∵ AC & BD bisect each other at

$\angle AOB = \angle COB = 90^\circ$ right angles

& $OB = OB$ {Common side

∴ By SAS Congruence Criterion

$\Delta AOB \cong \Delta COB$

$\Rightarrow$ $AB = CB$ (C.P.C.T)

But $AB = DC$ & $BC = AD$ {opp. sides of ∥gm)

∴ $AB = BC = CD = DA$

Now In ΔBAD & CDA, we have

$BA = CD$ (opp. sides of ∥gm

$AD = AD$ (Common

& $BD = AC$ (Given

∴ $\Delta BAD \cong \Delta CDA$ {By SSS Congruence Criterion.

$\Rightarrow$ $\angle A = \angle D$ (C.P.C.T

But, $\angle A + \angle D = 180^\circ$ (∵ Sum of Consecutive interior angles on the same side of transversal is 180°

$\Rightarrow$ $\angle A + \angle A = 180^\circ$

$2\angle A = 180^\circ \Rightarrow \angle A = \frac{\cancel{180}^{90}}{\cancel{2}} \Rightarrow \angle A = 90^\circ$

$\Rightarrow$ $\angle A = \angle D = 90^\circ$

Similarly, we have $\angle B = \angle C = 90^\circ$

∴ $\angle A = \angle B = \angle C = \angle D = 90^\circ$

Hence, ABCD is a square.

Q6:(i) ABCD is a Parallelogram. Diagonal AC bisects $\angle A$.
i.e, $\angle BAC = \angle DAC$ —— (1)

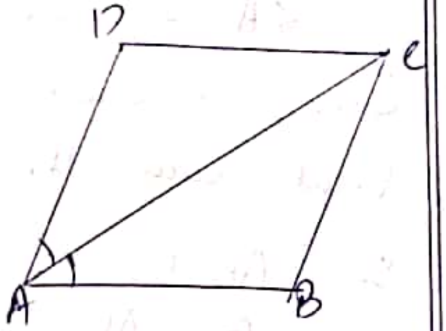
$\angle BCA = \angle DAC$ —— (2) (alt. $\angle$'s)

& $\angle DCA = \angle BAC$ —— (3) (alt. $\angle$'s)

From (1), (2) and (3)

$\angle BCA = \angle DCA$

$\Rightarrow$ AC bisects $\angle C$.



ii) From (1) and (2), in $\triangle BAC$, we have

$\angle BAC = \angle BCA$

$\Rightarrow$ $AB = BC$

Also, we have

$AB = CD$ and $BC = AD$ (Opposite Sides of a llgm)

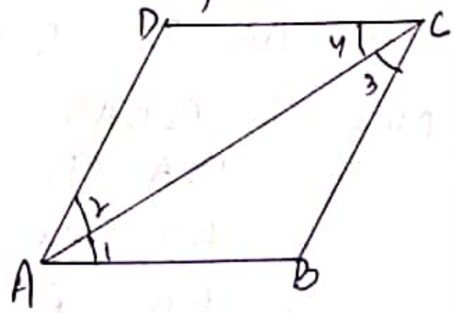
$\Rightarrow$ $AB = BC = CD = AD$

Hence, ABCD is a rhombus

Q7: Here, $\angle 1 = \angle 4$ —— (1) (Pair of alternate $\angle$'s)

In $\triangle ACD$, we have

$\angle 2 = \angle 4$ —— (2) ($\because AD = CD$)



From (1) and (2)

$\angle 1 = \angle 2$

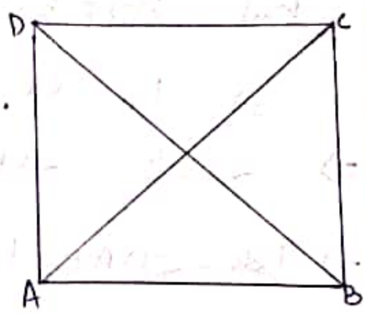
Similarly, $\angle 3 = \angle 4$

$\Rightarrow$ AC bisects $\angle A$ as well as $\angle C$

Illy, we can prove that, BD bisects $\angle B$ as well as $\angle D$.

Q8. Sol. i) let ABCD be a rectangle such that diagonal AC bisects $\angle A$ as well as $\angle C$ i.e,

$$\angle BAC = \angle DAC \;\&\; \angle BCA = \angle DCA$$



Since every rectangle is a Parallelogram.
Therefore,
$AB \parallel DC$ and AC is transversal

$\Rightarrow \angle BAC = \angle DCA$

But $\angle BAC = \angle DAC$ (Given

$\Rightarrow \angle DAC = \angle DCA$

$\Rightarrow DC = AD$ (Sides opp. to equal angles are equal

But $DC = AB$ & $AD = BC$ { opp. sides of rectangle

$\Rightarrow AB = BC = CD = DA$

Hence, ABCD is a square

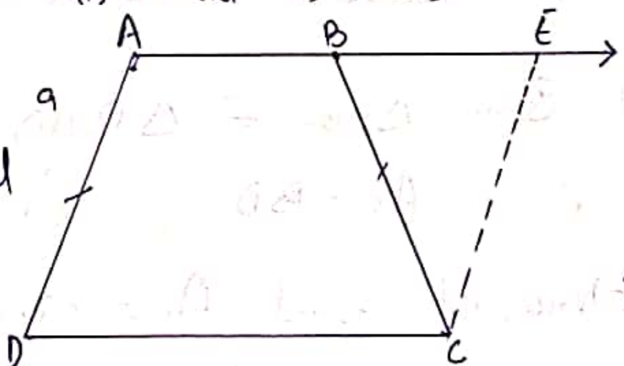
ii) In ΔBAD & ΔBCD, we have

$BA = CD$ } Sides of a square
$BC = AD$
& $BD = BD$ (Common)

$\therefore \Delta BAD \cong \Delta BCD$ { SSS Congruence Criterion

$\Rightarrow \angle ABD = \angle BCD$ and $\angle ADB = \angle CDB$

$\Rightarrow$ BD bisects $\angle B$ as well as $\angle D$.



Q12. (i) Extend AB and draw a line through C parallel to DA intersecting AB produced at E.

Since ABCD is a trapezium. Therefore, $AB \parallel CD$

Also, $DA \parallel CE$

So, ADCE is a Parallelogram

But

$\therefore$ $DA = CE$ and $DC = AE$

But $AD = BC$ (Given)

$\Rightarrow$ $BC = CE$

$\Rightarrow$ $\angle CEB = \angle CBE$ ($\angle$'s opp to equal sides are equal)

$\Rightarrow$ $\cancel{180^\circ} - \angle DAB = \cancel{180^\circ} - \angle ABC$ {$\because$ $\angle A + \angle E = 180^\circ$ as ADCE is a ||

$\Rightarrow$ $\angle DAB = \angle ABC$

$\Rightarrow$ $\angle A = \angle B$

ii) $\angle A + D = 180^\circ$ { $\angle$'s on the same side of the transversal are supplementary

& $\angle B + C = 180^\circ$

$\Rightarrow$ $\angle A + \angle D = \angle B + \angle C$

$\Rightarrow$ $\cancel{\angle A} + \angle D = \cancel{\angle A} + \angle C$ {$\because$ $\angle A = \angle B$

$\Rightarrow$ $\angle D = \angle C$

iii) In ΔABC and ΔBAD, we have

$AB = AB$ {Common

$\angle A = \angle B$ {Proved above

& $BC = BD$ (Given

$\Rightarrow$ $\Delta ABC \cong \Delta BAD$ {SAS Congruence

iv) Since $\Delta ABC \cong \Delta BAD$

$\therefore$ $AC = BD$ (C.P.C.T)

Hence, diagonal AC = diagonal BD

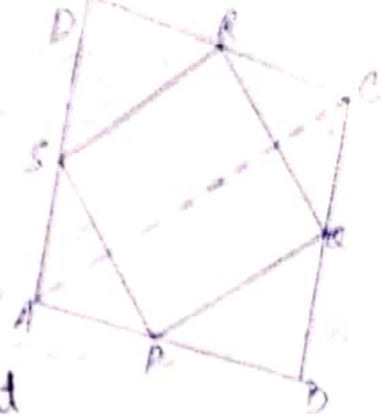
Exercise 7.2

i) In ΔDAC, S and R are the mid-points of the sides DA and DC respectively

$\therefore$ By mid-point theorem

$SR \parallel AC$ and $SR = \frac{1}{2} AC$

ii) In ΔBAC, P and Q are the mid-points of the sides AB and BC respectively



$\therefore$ By mid-point theorem

$PQ \parallel AC$ and $PQ = \frac{1}{2} AC$

$\therefore$ $PQ = SR$ {each $= \frac{1}{2} AC$

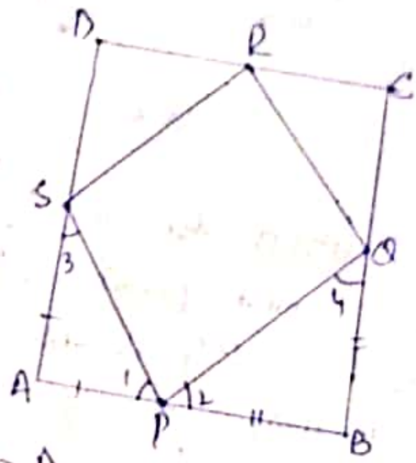
iii) we have already proved that

$SR \parallel AC$ and $PQ \parallel AC$ & $PQ = SR$

$\Rightarrow$ $SR \parallel PQ$ {If in a quad. pair of sides is $\parallel$ and equal then it is a $\parallel$gm.

$\Rightarrow$ PQRS is a Parallelogram

We can prove PQRS is a $\parallel$gm as in Q1



In ΔAPS, $AP = AS$

$\Rightarrow$ $\angle 1 = \angle 3$ {$\angle$'s opp. to equal sides are equal (3)

$\parallel$ly $\angle 2 = \angle 4$ - (4)

Also

$\angle 1 + \angle 3 + \angle A = 180°$ {A.S.P of Δ

$\Rightarrow$ $\angle 1 + \angle 3 = 180° - \angle A$

$\Rightarrow$ $\angle 1 + \angle 1 = 180° - \angle A$

$2\angle 1 = 180° - \angle A$ — (1)

$\parallel$ly $2\angle 2 = 180° - \angle B$ — (2)

Adding (1) & (2), we get

$2\angle 1 + 2\angle 2 = 180^\circ - \angle A + 180^\circ - \angle B$

$2(\angle 1 + \angle 2) = 360^\circ - \angle A - \angle B$

$2(\angle 1 + \angle 2) = 360^\circ - (\angle A + \angle B)$ { $\angle A + \angle B = 180^\circ$

$2(\angle 1 + \angle 2) = 360^\circ - 180^\circ$

$2(\angle 1 + \angle 2) = 180^\circ$

$\angle 1 + \angle 2 = \frac{180^\circ}{2}$

$\angle 1 + \angle 2 = 90^\circ$

Now

$\angle 1 + \angle 2 + \angle P = 180^\circ$ { $\angle$'s on the straight line

$\angle 1 + \angle 2 = 90^\circ$

$\therefore$ $90^\circ + \angle P = 180^\circ$

$\Rightarrow$ $\angle P = 180^\circ - 90^\circ$

$\Rightarrow$ $\angle P = 90^\circ$

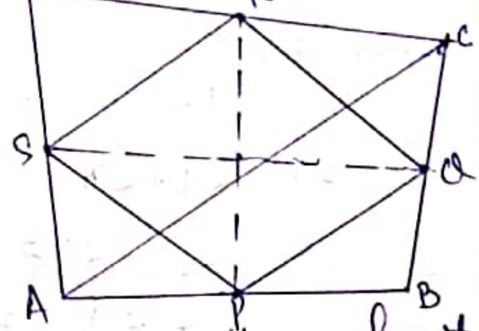
Illy, we can prove $\angle P = \angle Q = \angle R = \angle S = 90^\circ$

$\Rightarrow$ PQRS is a rectangle

- Let ABCD be a quadrilateral such that P, Q, R and S are the mid points of sides AB, BC, CD and DA respectively

In $\triangle ABC$, P & Q are mid points of AB & BC respectively

$\therefore$ $PQ \parallel AC$ & $PQ = \frac{1}{2} AC$



Illy, we have

$RS \parallel AC$ & $RS = \frac{1}{2} AC$

$\therefore$ $RS \parallel PQ$ { $\because$ both are $\parallel$ to AC

& $PQ = RS$ { $\because$ both are $= \frac{1}{2} AC$

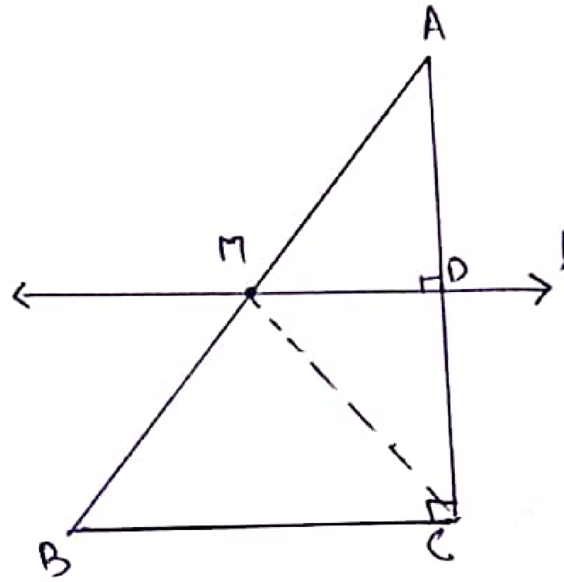
$\Rightarrow$ PQRS is a $\parallel$gm. Since diagonals of a $\parallel$gm bisect each other. Hence, PR & QS bisect each other.

7) i) Through M, we draw line $l \parallel BC$.
l intersects AC at D
$\Rightarrow$ D is mid-point of AC [by converse of mid-pt. theorem]

ii) $\angle ADM = \angle ACB = 90^\circ$

$\Rightarrow \angle ADM = 90^\circ$

$\Rightarrow MD \perp AC$



iii) In ΔCMD & ΔAMD;
$CD = AD$, $MD = MD$
and $\angle CDM = \angle ADM$ (each 90°)

Therefore, $\Delta CMD \cong \Delta AMD$

$\Rightarrow CM = AM$
Also, $AM = \frac{1}{2} AB$ {$\because$ M is mid pt. of AB
$\Rightarrow CM = AM = \frac{1}{2} AB$

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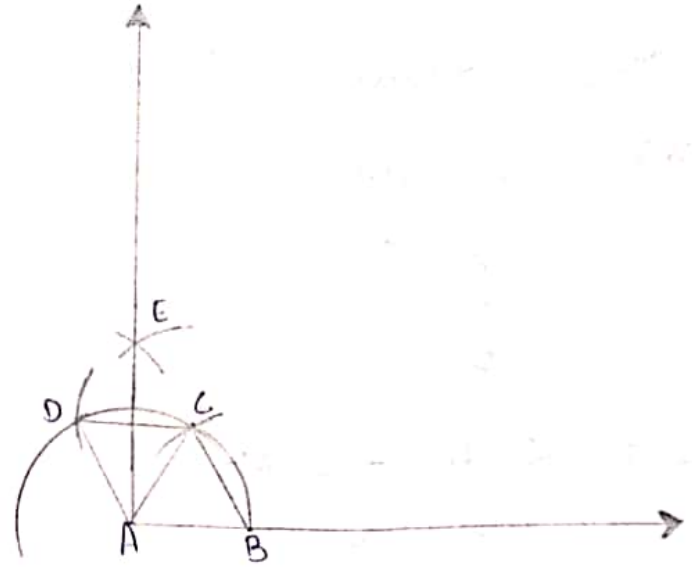
Constructions

Exercise 10.1



Steps Of Construction

1) Let A be the initial point of a given ray.
2) With 'A' as Centre, draw an arc of any radius which intersects the ray at B.
3) With 'B' as Centre and the same radius as before draw an arc which cuts previous arc at point C.
4) With 'C' as Centre and same radius draw another arc which cuts first arc at 'D'.
5) With 'C' and 'D' as Centres and radius more than half of CD, draw two arcs intersecting each other at E.
6) Join AE.

$\angle EAB$ is the required angle of measure 90°.

Justification

Join AC, BC, AD & CD

Clearly $AB = AC$ {Radii of same arcs

$AD = AC$ {Radii arcs of same radii

$\Rightarrow AB = BC = AC$

$\Rightarrow \Delta ABC$ is an equilateral Δ

$\therefore \angle CAB = 60^\circ$

IIly, we can prove ΔADC is an equilateral Δ

$\therefore \angle DAC = 60^\circ$

Now, ray AE bisects $\angle DAC$

$\therefore \angle EAC = \angle DAE = \frac{1}{2} \angle DAC$

$= \frac{1}{\cancel{2}} \times \cancel{60}^{30^\circ}$

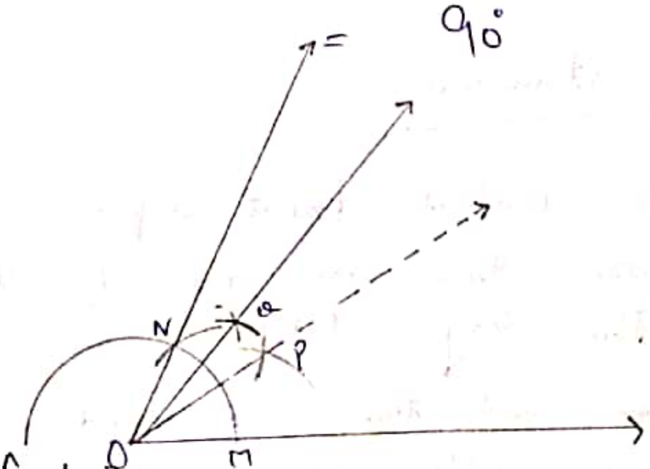
$= 30^\circ$

Hence, $\angle EAB = \angle EAC + \angle CAB$

$= 30^\circ + 60^\circ$

$= 90^\circ$

Q2:



Steps of Construction:

1) Let 'O' be the initial point of a given ray.
2) With 'O' as centre draw an arc intersecting the ray at M.
3) With 'M' as centre and same radius draw an arc which cuts previous arc at N
4) Draw OP angle bisector of $\angle MON$ s.t $\angle NOP = \angle MOP = 30^\circ$
5) Draw OQ angle bisector of $\angle NOP$ s.t $\angle POQ = 15^\circ$
6) $\angle MOQ$ is the required angle of measure 45°.

Justification:

Join NM

Now In ΔONM

ON = OM (radii of same radii)

OM = MN (radii of same radii)

$\Rightarrow \Delta ONM$ is an equilateral Δ

$\therefore \angle MON = 60^\circ$

Now OP bisects $\angle MON$

$\angle NOP = \angle MOP = \frac{1}{2} \angle MON = \frac{1}{\cancel{2}} \times \cancel{60}^{30}$

$= 30^\circ$

Now OQ bisects $\angle NOP$

$\angle POQ = \frac{1}{2} \angle NOP = \frac{1}{\cancel{2}} \times \cancel{30}^{15}$

$= 15^\circ$

$\angle MOQ = \angle MOP + \angle POQ$

$= 30^\circ + 15^\circ = 45^\circ$

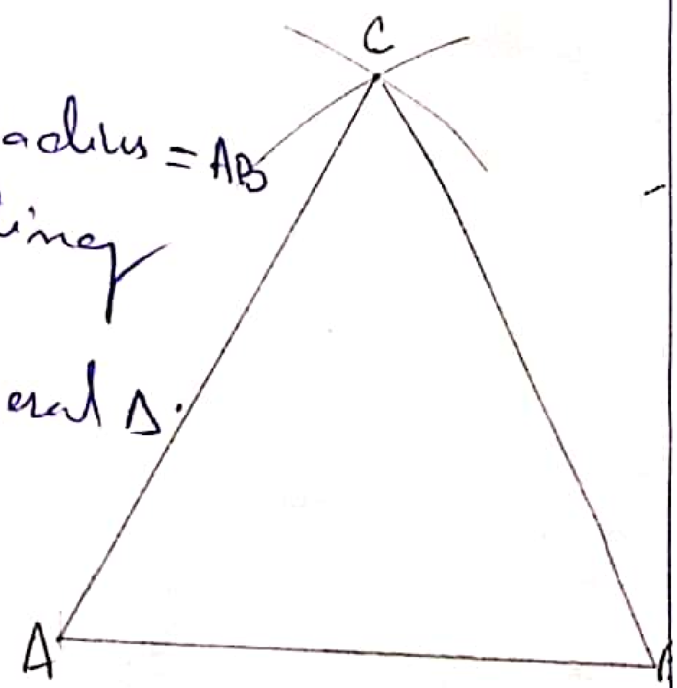
Q5.

Steps of Construction

Draw a line segment $AB = 6cm$.
with 'A' and 'B' as Centres & radius = AB
= 5cm. draw two arcs intersecting each other at C.
$\triangle ABC$ is the required equilateral $\triangle$.

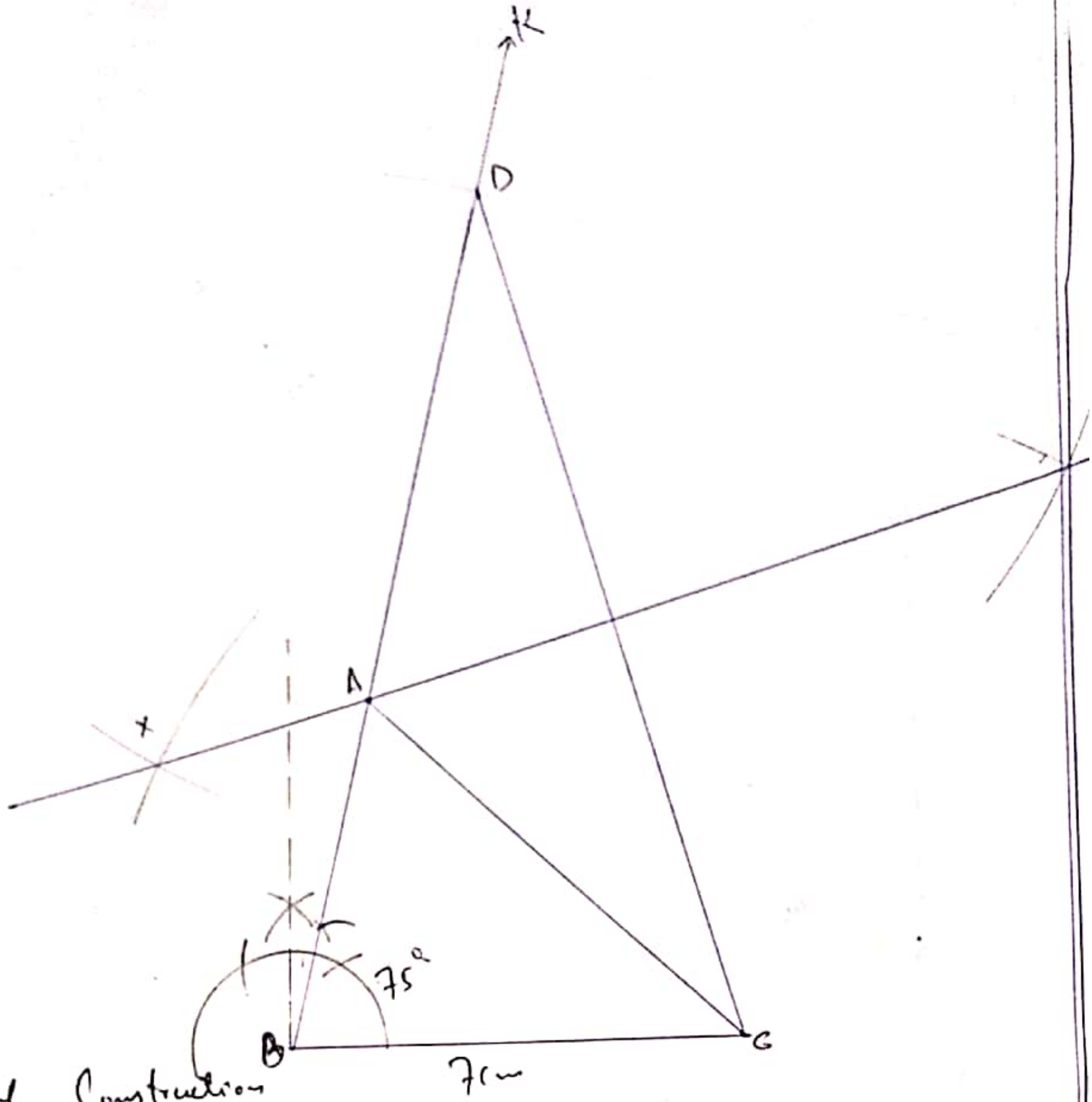
Justification

$AC = BC = 5cm$ {arcs of same radii
$AB = BC = AC = 5cm$
$\Rightarrow$ $\triangle ABC$ is an equilateral $\triangle$.



Exercise 10.2

Q1



Steps of Construction

i) Draw a line segment BC = 7cm

ii) Draw ray $\overrightarrow{BK}$ s.t $\angle CBK = 75°$

iii) Cut BD = 13cm out of BK.

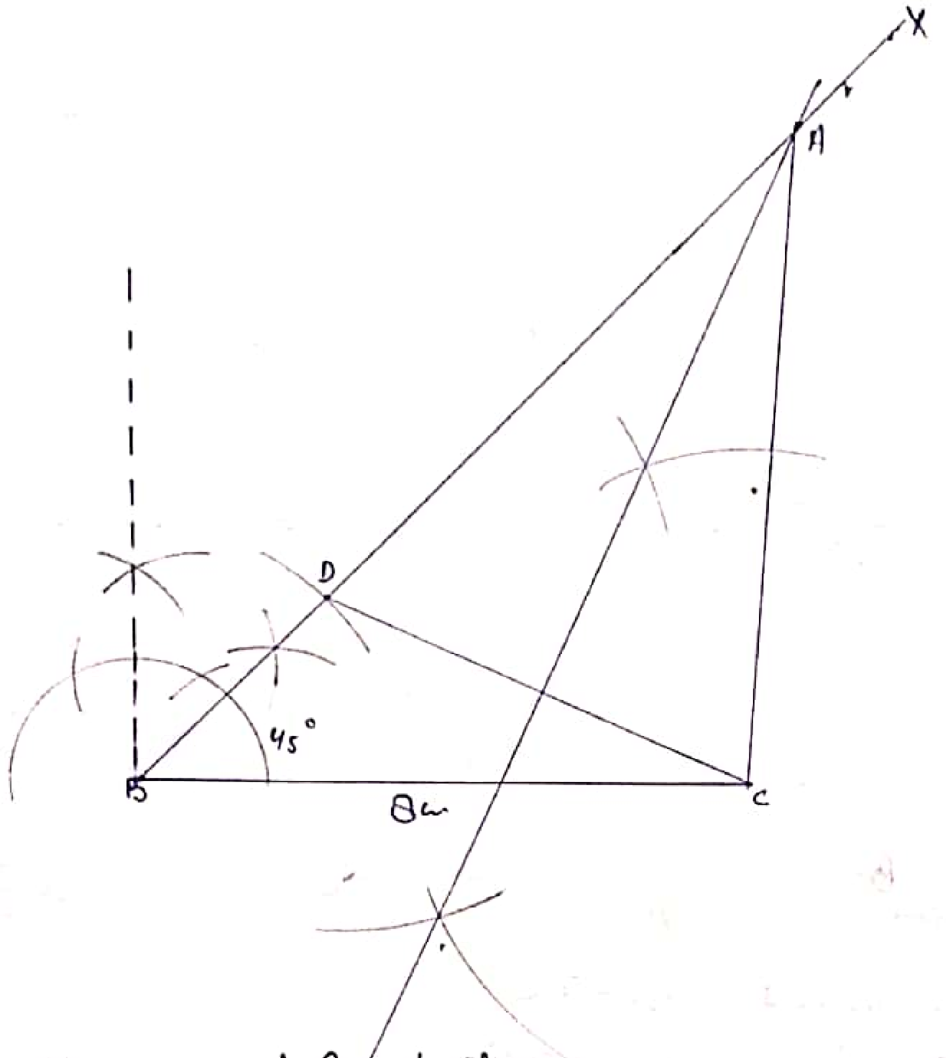
iv) Join CD.

v) Draw XY right bisector of CD intersecting BD at A

vi) Join CA.

Now ΔABC is the required Δ.

Q2:



Steps of Construction

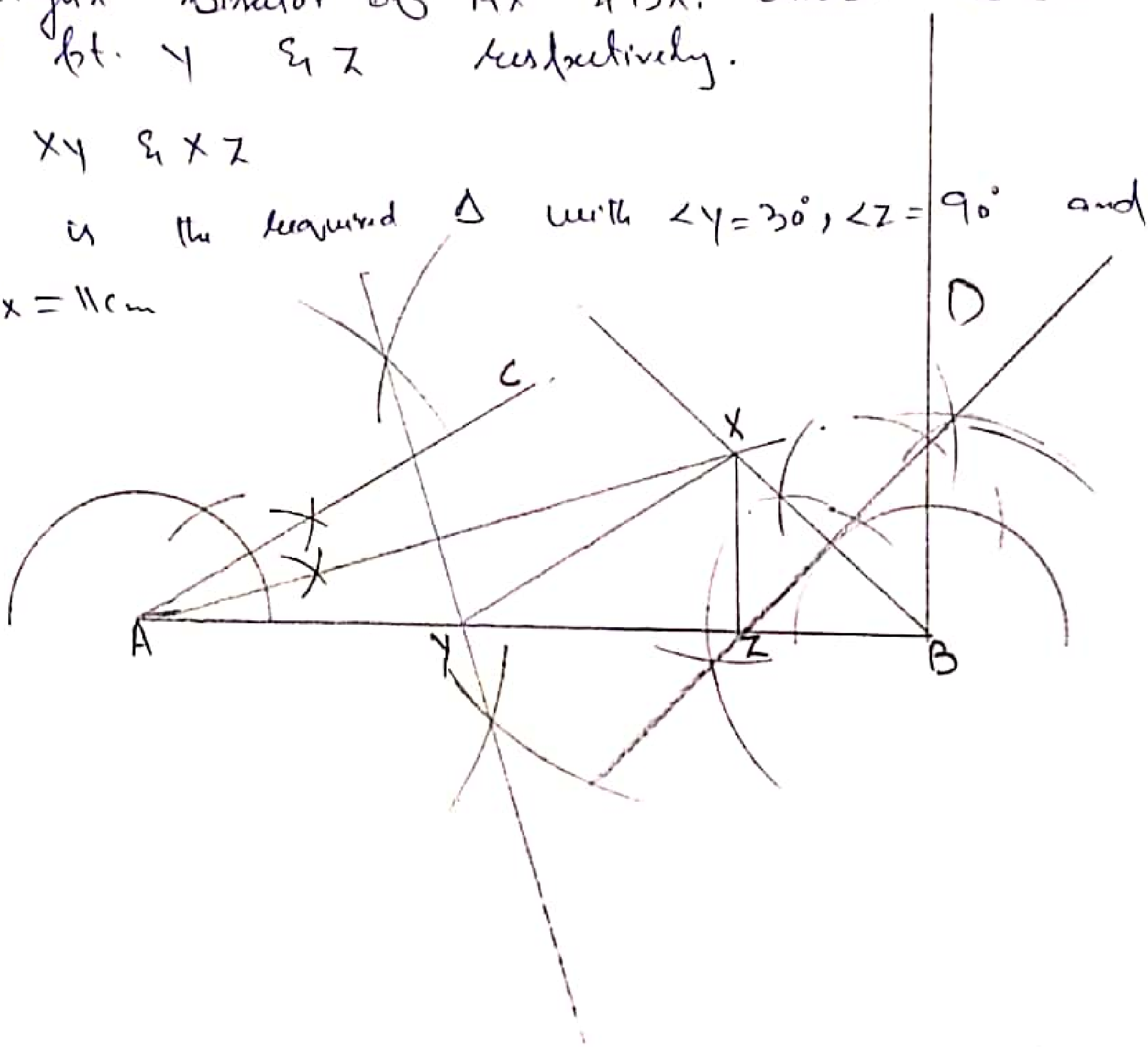
i) Draw a line segment BC = 8cm
ii) At B, make an angle <XBC = 45°
iii) From BX cut BD = 3.5cm & Join CD
iv) Draw bisector of CD which cuts BX at A
v) Join AC
ΔABC is the required Δ.

Q4

Steps of Construction

1) Draw line segment AB = 11cm
2) At A, make ∠CAB = 30°
3) At B, make ∠ABD = 90°
4) Bisect ∠CAB & ∠ABD, let the angle bisectors meet at pt. X
5) Draw right bisector of AX & BX, which cut AB at pt. Y & Z respectively.
6) Join XY & XZ

ΔXYZ is the required Δ with ∠Y = 30°, ∠Z = 90° and XY + YZ + ZX = 11cm



Statistics

Statistics: Statistics is the branch of mathematics, which deals with the Collection, analysis and interpretation of numerical data.

On the basis of Collection, data are of two types

Primary data: The data Collected actually in the process of investigation by the investigator is Called primary data. It is Original and first hand information.

Secondary data: The data Collected by someone and used by any other person is known as Secondary data.

Raw Or Ungrouped data: when the data presented is random and is not prepared according to some Order, it is known as raw Or Ungrouped data. It doesnot give us Clear picture of the Class.

Grouped data: when the data is arranged in any manner like ascending Or descending Order etc. it is Called grouped data. It Can be presented in the form of table Called frequency distribution table.

Class intervals: Class intervals are the groups in which all the observations are divided. Each class is bounded by two figures (numbers) which are called class limits. The figure on the left side of a class, is called its lower limit and that on the right side of a class is called upper limit.

Class Mark: It is the mid-point of the class interval
i.e, $\text{Class Mark} = \dfrac{\text{lower class limit} + \text{upper class limit}}{2}$

Range or a Class Size: Difference between the upper limit and the lower limit of a class is called its class size.

i.e Range = upper limit − lower limit

Frequency of an Observation: The number of times an observation occurs is called its frequency.

Graphical Representation of Data

Bar Graph: A bar graph is a pictorial representation of the numerical data by a series of bars or rectangles of uniform width standing on the same horizontal (or vertical) base line with equal spacing between the bars.

Histogram: This is a graphical form of representation but it is used for continuous class intervals.

Frequency Polygon: A frequency polygon is a graph constructed by using lines to join the mid points (class marks) of each interval. The heights of the points represent the frequencies.

Arithmetic Mean: If $x_1, x_2, x_3, \dots, x_n$ are n values of a variable X, then the arithmetic mean or simply the mean of these values is denoted by $\bar{X}$ and is defined as

$$\bar{X} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)$$

Here, the symbol $\sum_{i=1}^{n} x_i$ denotes the sum $x_1 + x_2 + \dots + x_n$

In otherwords, the arithmetic mean of a set of observations is equal to their sum divided by the total no. of observations.

Median: Median of a distribution is the value of the variable which divides the distribution into two equal parts i.e it is the value of the variable such that the number of observations above it is equal to the number of observations below it.

If no. of observations (n) is odd, then

$$\text{Median} = \text{value } \left(\frac{n+1}{2}\right)^{th} \text{ observation}$$

If n is is even, then

$$\text{Median} = \frac{\text{Value of } \left(\frac{n}{2}\right)^{th} \text{observation} + \text{value of } \left(\frac{n}{2}+1\right)^{th} \text{ob}}{2}$$

Mode: Mode is the value which occurs most frequ in a set of observations.

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Exercise 13.2

Q1.

Blood group	Tally Marks	Number of students
A	卌 ||||	9
B	卌 |	6
O	卌 卌 ||	12
AB	|||	3
Total		30

Most Common blood group is 'O' and the rarest blood group is 'AB'

Q2.

Distances (in km)	Tally Marks	Frequency
0-5	卌	5
5-10	卌 卌 |	11
10-15	卌 卌 |	11
15-20	卌 ||||	9
20-25	|	1
25-30	|	1
30-35	||	2
Total		40.

Q3: i)

Relative Humidity (in %)	Frequency
84 - 86	1
86 - 88	1
88 - 90	2
90 - 92	2
92 - 94	7
94 - 96	6
96 - 98	7
98 - 100	4
Total	30

(ii) The data appears to be taken in the rainy season as the relative humidity is high.

(iii) Range = 99.2 - 84.9 = 14.3

Q5: i)

Concentration of Sulphur dioxide (in ppm)	Frequency
0.00 - 0.04	4
0.04 - 0.08	9
0.08 - 0.12	9
0.12 - 0.16	2
0.16 - 0.20	4
0.20 - 0.24	2
Total	30

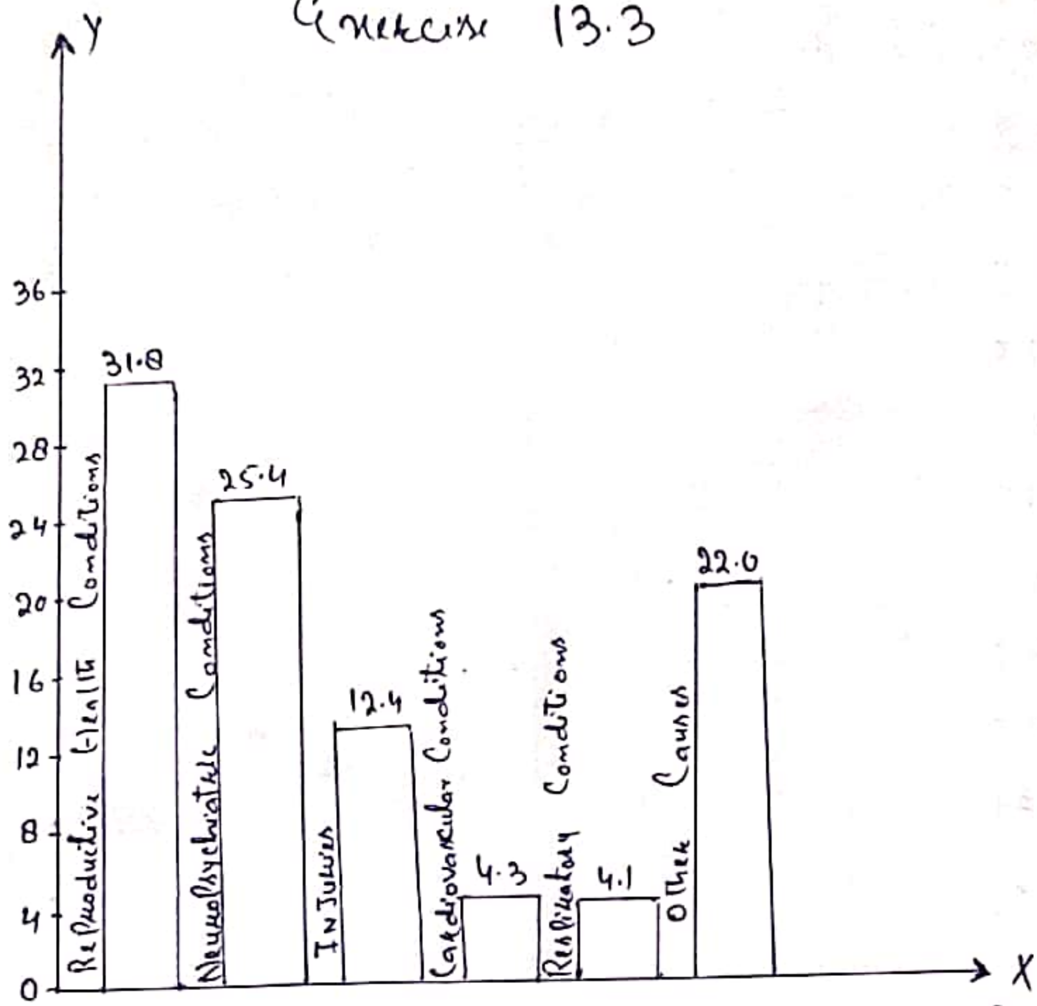
ii) The Concentration of Sulphur dioxide was more than 0.11 ppm for (2+4+2) = 8 days

Q7: (i)

Digits	Frequency
0	2
1	5
2	5
3	8
4	4
5	5
6	4
7	4
8	5
9	8
Total	50

Exercise 13.3

Q1: (i)



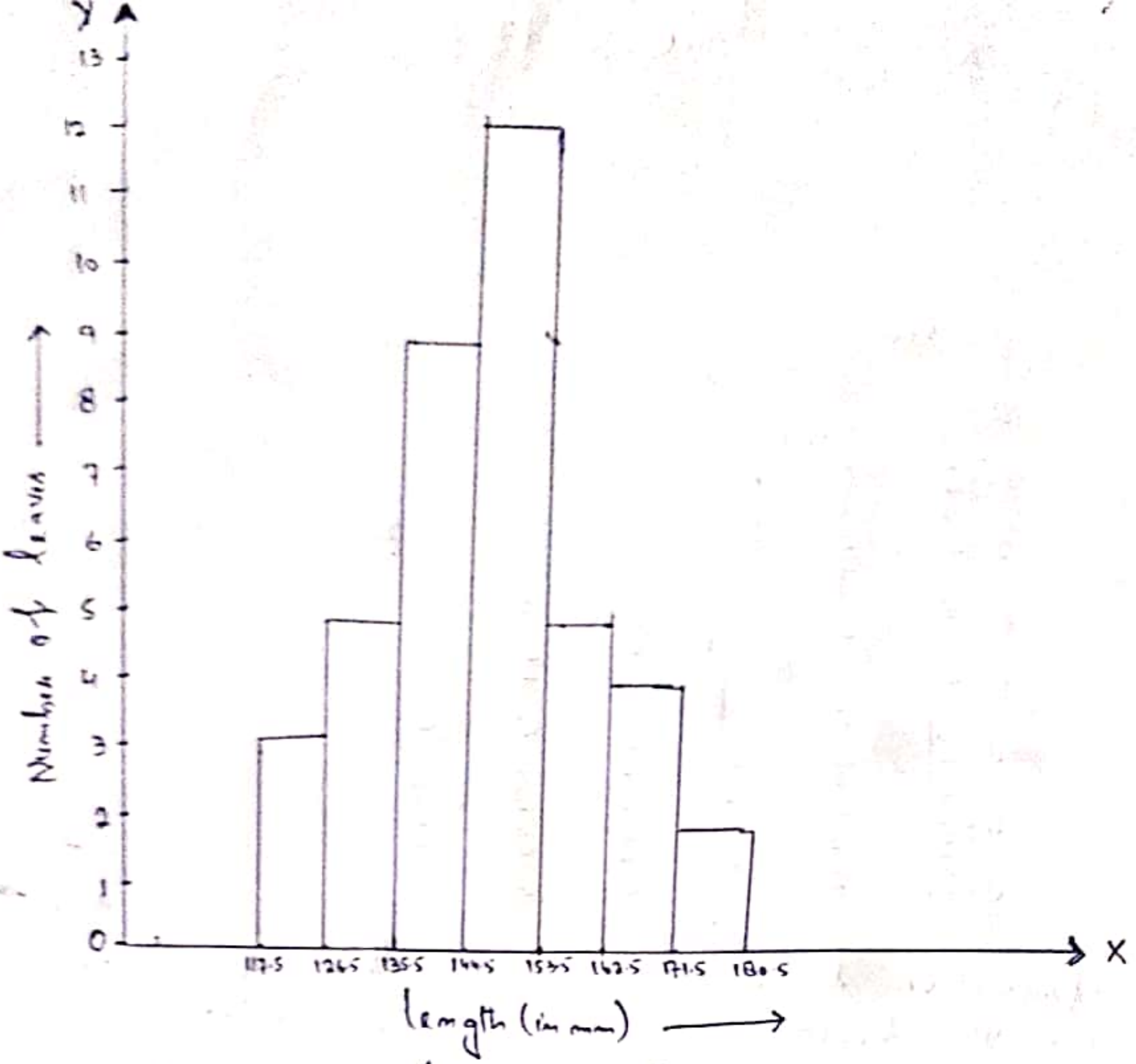
ii) Reproductive health Condition is the major Cause of women's ill health and death worldwide.

Q4:

i)

length (in mm)	Number of leaves
117.5 – 126.5	3
126.5 – 135.5	5
135.5 – 144.5	9
144.5 – 153.5	12
153.5 – 162.5	5
162.5 – 171.5	4
171.5 – 180.5	2

(i)



ii) Yes, we can make frequency polygon to represent the data.

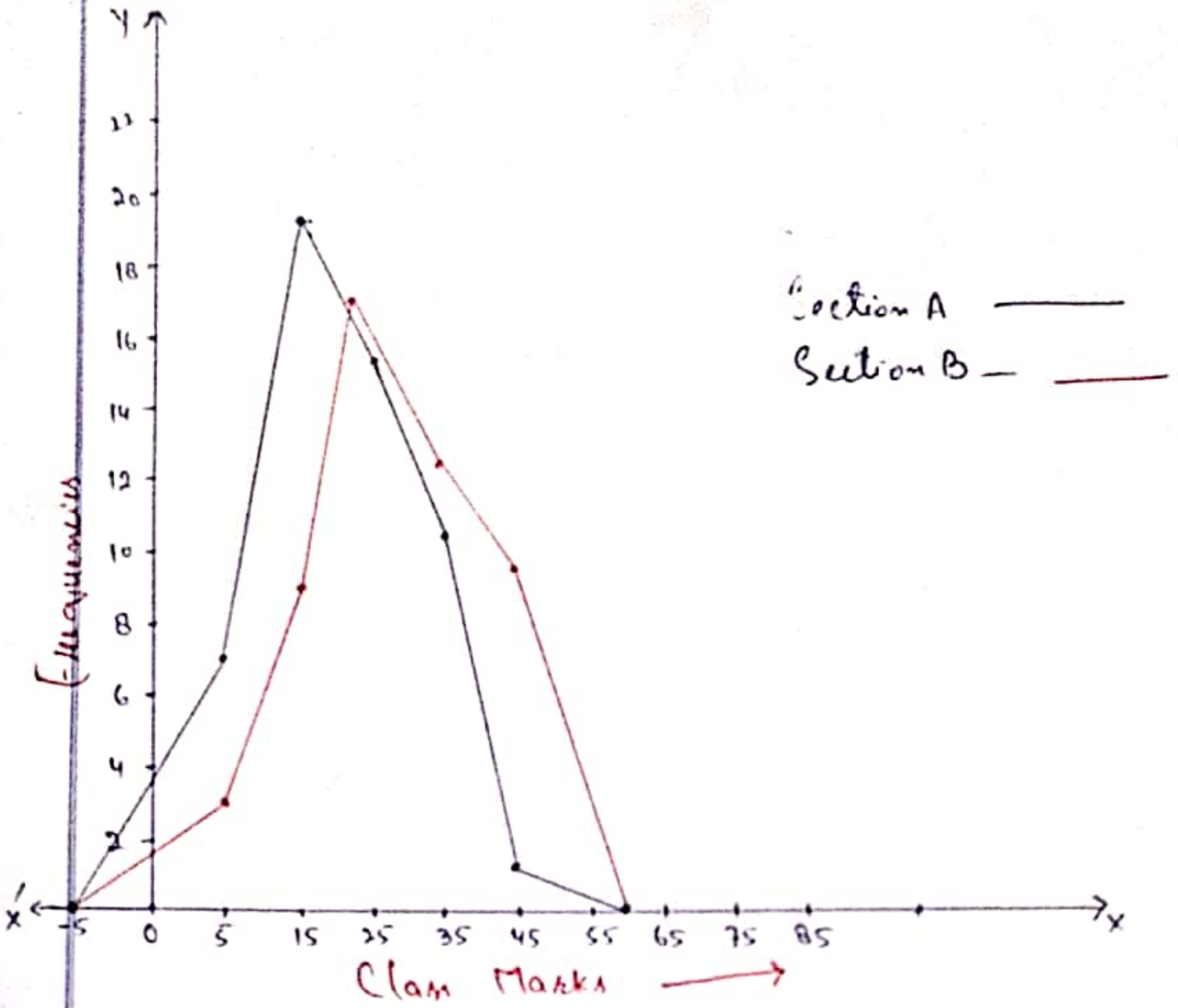
iii) No, the no. of leaves having length 153cm or less than 153cm $= 3+5+9+12 = 29$

and the no. of leaves having length ~~153cm~~ more than 153cm $= 5+4+2 = 11$

Q6

Section A		
Marks	Frequency	Class-Mark
0 – 10	3	$\frac{0+10}{2} = \frac{10}{2} = 5$
10 – 20	9	$\frac{10+20}{2} = \frac{30}{2} = 15$
20 – 30	17	$\frac{20+30}{2} = \frac{50}{2} = 25$
30 – 40	12	$\frac{30+40}{2} = \frac{70}{2} = 35$
40 – 50	9	$\frac{40+50}{2} = \frac{90}{2} = 45$

Section B		
Marks	Frequency	Class-Mark
0 – 10	5	$\frac{0+10}{2} = \frac{10}{2} = 5$
10 – 20	19	$\frac{10+20}{2} = \frac{30}{2} = 15$
20 – 30	15	$\frac{20+30}{2} = \frac{50}{2} = 25$
30 – 40	10	$\frac{30+40}{2} = \frac{70}{2} = 3$
40 – 50	1	$\frac{40+50}{2} = \frac{90}{2} = ($


Y
22
20
18
16
14
12
10
8
6
4
2
Frequencies
X'
-5
0
5
15
25
35
45
55
65
75
85
X
Class Marks
Section A
Section B

Ex. 14.4

Q1) The data has 10 values. we arrange these values in the ascending order below:
0, 1, 2, 3, 3, 3, 3, 4, 4, 5.

i) $\text{Mean} = \dfrac{\text{Sum of observations}}{\text{No. of observations}}$

$$= \frac{0+1+2+3+3+3+3+4+4+5}{10}$$

$$= \frac{28}{10} = 2.8$$

Mean = 2.8

(ii) Since the no. of observations (n) is even (10)

i. $\text{Median} = \dfrac{\left(\frac{n}{2}\right)^{th}_{ob.} + \left(\frac{n}{2}+1\right)^{th} ob.}{2}$

$$= \frac{\left(\cancel{\frac{10}{2}}^{5}\right)^{th} ob + \left(\cancel{\frac{10}{2}}^{5}+1\right)^{th} ob.}{2}$$

$$= \frac{5^{th}\ ob. + 6^{th}\ ob.}{2}$$

$$= \frac{3+3}{2} = \cancel{\frac{6}{2}}^{3}$$

$$= 3$$

Therefore, Median = 3

First, Arrange the data in ascending order

39, 40, 40, 41, 42, 46, 48, 52, 52, 52, 54, 60, 62, 96, 98

Here, no. of values $(n) = 15$

Then, Mean $= \frac{\text{Sum of Ob.}}{\text{no. of ob.}}$

$$= \frac{39+40+40+41+42+46+48+52+52+52+54+60+62+96+98}{15}$$

$$= \frac{822}{15} = 54.8$$

$\therefore$ Mean $= 54.8$

ii) Since, $n = 15$ (i.e, Odd)

$\therefore$ Median $= \left(\frac{n+1}{2}\right)^{th}$ ob.

$$= \left(\frac{15+1}{2}\right)^{th} \text{ ob} = \left(\frac{\cancel{16}^{8}}{\cancel{2}}\right)^{th} \text{ ob}$$

$$= 8^{th} \text{ ob.}$$

8^{th} Ob. in the arranged data is 52

$\therefore$ Median $= 52$

iii) Mode $= 52$ ($\because$ 52 occurs most of the times with frequency 4)

The data is already arranged in ascending order

29, 32, 48, 50, x, $x+2$, 72, 78, 84, 95

Median = 63

Here, no. of ob. $(n) = 10$

n is even

$\therefore$ Median $= \dfrac{\left(\frac{n}{2}\right)^{th} \text{ob} + \left(\frac{n}{2}+1\right)^{th} \text{ob.}}{2}$

$63 = \dfrac{\left(\frac{\cancel{10}^{5}}{\cancel{2}}\right)^{th} \text{ob} + \left(\frac{\cancel{10}^{5}}{\cancel{2}}+1\right)^{th} \text{ob.}}{2}$

$63 = \dfrac{5^{th} \text{ ob.} + 6^{th} \text{ ob.}}{2}$

$63 = \dfrac{x+x+2}{2}$ (5th ob. in arranged data is x & 6th ob. is $x+2$

$63 \times 2 = 2x+2$

$126 = 2x+2$

$126-2 = 2x$

$124 = 2x$

$\dfrac{\cancel{124}^{62}}{\cancel{2}} = x$

$62 = x$

or $\boxed{x = 62}$

. Mode of the given data is 14 with maximum frequency of 4

Salary (in Rs)	No of workers	$f_i x_i$
3,000	16	$3{,}000 \times 16 = 48{,}000$
4,000	12	$4{,}000 \times 12 = 48{,}000$
5,000	10	$5{,}000 \times 10 = 50{,}000$
6,000	8	$6{,}000 \times 8 = 48{,}000$
7,000	6	$7{,}000 \times 6 = 42{,}000$
8,000	4	$8{,}000 \times 4 = 32{,}000$
9,000	3	$9{,}000 \times 3 = 27{,}000$
10,000	1	$10{,}000 \times 1 = 10{,}000$
Total	$\sum_{i=1}^{8} f_i = 60$	$\sum_{i=1}^{8} f_i x_i = 3{,}05{,}000$

$$\therefore \text{Mean } (\bar{x}) = \frac{\sum_{i=1}^{8} f_i x_i}{\sum_{i=1}^{8} f_i} = \frac{3{,}05{,}000}{60} = \text{Rs } 5083.33$$